Teaching Mathematics with Digital Content Resources (DCR) in Pakistan. A Review of Emerging Frameworks in Mathematics Education Research

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Abstract

In recent years, the eLearning industry in Pakistan has been engaged in the design and development of digital content resources (DCR) to support the teaching and learning of school mathematics. However, little effort has been made to examine the current knowledge and skills of high school mathematics teachers for the use of DCR. In contrast, a growing interest of international scholars is noticeable in the field of "Resources for Mathematics Teachers" which also includes digital content resources. Various theoretical, methodological and conceptual frameworks have emerged with this concern. This paper examined recent studies for the use of DCR in mathematics education and identified frameworks such as TPACK (technological pedagogical and content knowledge), PTK (pedagogical technological knowledge), Instrumental Genesis, and MDKT (mathematical digital knowledge for teaching) that can be used by national (Pakistan) and international researchers as a lens to study the knowledge and skills of high school mathematics teachers for the use of DCR.

Keywords: *Mathematics education, Teaching, Digital content resources, Digital competences, Digital technology, Knowledge and skills, Instrumental genesis, Framework*

1. Introduction

Digital content resources (DCRs) such as videos, images, graphics, animations, interactive games and infographics make it possible for mathematics teachers to include a variety of resources, representations and real-world ideas in their lessons. Teachers' reliance on digital content resources to develop a mathematics curriculum is increasing globally (Pepin, Gueudet, & Trouche, 2017). Studies have shown that DCR can be used as a tool to improve Mathematics pedagogy (Choppin, Carsons, Bory, & Cerosaletti, 2014; Gaffney, 2010; Tabach & Trgalová, 2019). It is now widely

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accepted that digital tools and technologies can contribute in establishing the vital cognitive connections, enhance active participation and recognition of mathematical concepts (Gueudet, 2015; Keller, Hart, & Martin, 2001; Lee & Hollebrands, 2008; Pepin, Choppin, Ruthven, & Sinclair, 2017). Apart from classroom levels, the importance of using digital tools and technologies in Mathematics education is growing at National levels as well. Several online platforms (just to name a few, such as; Mexico's Enciclomedia, Italy's M@t.abel; the US's Sketchpad for Young Learners, Lithuania's Mathematics 9 and 10 with The Geometer's Sketchpad, European Union Edumatics, ArAl (Arithmetic and Algebra) Project, and Iran's E-content initiative) have evolved recently to cater the need of digital content resources in mathematics. The list surely becomes more exhaustive when we consider private or not for profit digital content highly publicised platforms such as Khan Academy, Math is Fun, Coursera, Edx, IXL New Zealand and some others that emerged as the results of significant capital investment for example publishers' efforts (Pepin, Choppin, et al., 2017).

Encouragingly, during the past few years, the eLearning industry in Pakistan has also started to engage with the development of digital content resources. These digital resources are largely bilingual educational videos streamed through static websites, and a few platforms also have the facility to mediate online notes and e-textbooks (Raza, 2016). Teachers have begun using generic and free DCR, particularly in urban high schools. However, the National Curriculum of Pakistan does not provide any particular set of guidelines for the use of DCR. Teachers select and use digital resources based on their knowledge, experiences and skills. Whereas studies (Akhter, Akhtar, & Abaidullah, 2015; Aslam & Kingdon, 2011; Kanwal, Rehman, Bashir, & Qureshi, 2017; Khalil, 2016; Mohyuddin, 2012; Nordin, Zaman, & Din, 2005; Raza, 2016) have shown that teachers in Pakistan have limited knowledge to use digital technologies for educational purposes and little efforts are made to improve such knowledge. Therefore, it will be interesting to investigate how teachers in Pakistan are integrating digital content resources for teaching and learning of mathematics.

Besides, the analysis of literature for mathematics education in Pakistan revealed that most of the investigations in the field are student-centric (cf. Akhter et al., 2015; Ministry of Education Curriculum Wing, 2007; Mohyuddin, 2012; Redden et al., 2010; Rehman, 2011; Suleman, Aslam, & Hussain, 2014; Tayyaba, 2010; Zaman, Farooq, Hussain, Ghaffar, & Satti, 2017b). Researchers have made limited attempts to examine the use of DCR by teachers under frameworks of mathematics education. Further search using the Higher Education Commission of Pakistan (HEC) research repository with the search criteria 'mathematics AND education', identified 24 dissertations (1960 – 2019). Only five of these used 'technology' in their title with no study discussing digital content resources. Also, the search results of academic databases (ProQuest, ERIC via ProQuest, ScienceDirect, EBSCO, Scopus, and JSTOR) were unable to locate studies that investigate the use of digital content resources by high school mathematics teachers in Pakistan. This shows a knowledge gap in Pakistan's existing literature on mathematics education. Therefore, at first, this article will identify how DCR is defined in the literature. Second, the article will discuss, how teachers around the globe integrating digital content resources and what complexities involved in the use of DCR for mathematics education. Then finally, the article will attempt to identify theoretical and methodological frameworks (such as; TPACK, PTK, double instrumental genesis and Digital Competencies framework) which can be employed as a lens to study the teaching of mathematics with digital content resources in Pakistan.

2. Literature Review

2.1 Defining Digital Content Resources (DCR)

The term DCR is used synonymously with digital content materials or digital curriculum resources that can be used in a variety of formats by teachers. Mullan (2011) defines DCR as all that is publishable on the Internet. Intel Corporation (2011) provides details of common components of DCR: "Digital content that can serve the purpose of education is referred to as content in the form of multimedia components such as images, graphics, infographics, audios, videos, texts, animations, simulations, interactive and gaming resources" (p. 2). Gaffney (2010), however, refers digital content resources or “digital curriculum resources to online curriculum content that can be used and customised by teachers in a variety of formats, including interactive multimedia resources, interactive assessment resources, and digital curriculum resources, which have been sourced from cultural and scientific institutions and private collections" (p. 4). Gaffney's referring of DCR to digital curriculum resources resonates with Pepin, Choppin, et al., (2017) description of digital content resources in which they describe curriculum resources/materials as an elastic concept starting "from one-off worksheets to a full range of curriculum schemes/programmes" (p. 647). Pepin, Choppin, et al. (2017) suggest while defining DCR focus should be on resources in digital formats that can organise and "articulate a scope of curricular content as per age, level, grade, topic, and content-wise" (p. 647). In a nutshell, the above definitions imply, the term DCR is flexible, any resource, application, digital space, or multimedia content (available freely or through a subscription) which can be used for teaching and learning may be considered as DCR.

2.2 Digital content resources (DCR) for mathematics education

There is no agreed definition of what constitutes DCR for mathematics. However, given above definitions, the DCR for mathematics education may be defined as an educational digital content that is available both online and offline, contain multimedia components (as defined by Intel Corporation), dynamic features which are designed, developed, customised, used and updated by the mathematics community (teachers, students, institutions, developers and others). The definition includes multimedia, as Mayer (2003) suggests that multimedia components can involve students in learning more deeply than a single communication process such as print materials. However, Mayer (2003) suggests different media, notably text and visuals, function best when they are close together and contain no 'extraneous' details. In addition to the multimedia components, tools such as; graphs, calculators, surveys, spreadsheets, wikis, academic networking with students and experts are available to further enhance DCR (Choppin et al., 2014; Nicholas & Lewis, 2011) for mathematics education. Also, DCR is likely to support multiple dialects, adequate feedback and answer mechanism, an easy to keep up-to-date, subscription-based, search-and-sort, select, contain exporting, reformatting and combining text and other content options for teaching and learning (Intel Corporation, 2011).

2.3 Integration of Digital Content Resources (DCR) in Mathematics Teaching

In this section, few studies are presented that have examined the integration of instructional resources (DCR) and decisions made by teachers when finding and selecting DCR for mathematics teaching and learning. For example, Esguerra-Prieto, González-Garzón and Acosta-López (2018) have shown that complex numbers can be taught in a simplified way by creating graphics using MATLAB and GeoGebra. They used MATLAB and GeoGebra to create (3D representations) and perform several operations of complex numbers graphically such as addition, multiplication, division, subtraction, and conjugate. Maria, Manuel, Santos and Santos (2015) also demonstrated the use of GeoGebra to study complex numbers and complex functions. They produced multiple representations of complex numbers using 2D and 3D graphic windows in GeoGebra to solve and teach advance mathematical concepts in complex numbers. In addition to complex numbers, both software can be used for teaching several other mathematical topics. For example, Khalil (2016) in experimental study design (posttest equivalent group) investigated the result of GeoGebra on high school students' mathematical thinking and achievement in analytical geometry. The results of the study had shown that experimental group students performed better when they were taught analytical geometry using GeoGebra.

Gueudet (2015) investigated mathematics teachers' work with resources using an approach she called documentational approach. Gueudet argued that a teacher interacts with both old and new resources to achieve specific pedagogical goals. She explains 'old resources' as resources that are already appropriated, whereas 'new resources' are those which are often found on the Internet, "or selected or designed by colleagues, or presented in in-service training sessions" (Trouche et al., 2018, p. 5). The Documentational approach considers these multiple interactions of teachers with resources results in a document. Gueudet argued that the use of digital resources is a multi-stage process in which a teacher goes through several stages of learning. Gueudet (2015) using an inquiry-based-activity approach established that digital representations of graphs could be both interesting and confusing at the same time. It all depends on how teachers use, select, integrate and analyse digital resources in combination with the other resources they have.

Maher, Palius, Maher, Hmelo-Silver, and Sigley (2014) considered videos as an essential tool for in-service and pre-service mathematics teachers' professional development. They showed videos could facilitate teachers in identifying patterns of students' mathematical reasoning; in improving their engagement with students in mathematical discussions, construction of mathematical argument and how to critique the reasoning of others. Similarly, Arzarello and Sabena (2014) used video recording of two students working on a mathematical problem (exponential functions) to understand how students develop mathematical concepts. They used Action Production and Communication (APC) Theory framed under the Vygotskian perspective of social constructivism for investigation. They found videos very useful in doing microanalysis of students' gestures, which they consider a useful way of thinking and communicating in co-constructing mathematical concepts. de Araujo, Otten, and Birisci (2017) presented a case of a mathematics teacher who created and used videos (DCR) as an alternative to the textbook in a mathematics classroom. She scripted the video as per the content of the textbook, enriched it with her imagination and representation. This use of digital content solved her problem of teaching and attending to her students at the same time.

The study by Yerushalmy, Nagari-Haddif, and Olsher (2017) used online interactive assessment content (DCR) to understand the meaning of high school mathematics students' submission of answers. The objective of the study was to reduce the time teachers spend in understanding and interpreting students' responses by using the auto feedback mechanism provided by the online system. The results of the study showed that the use of digital technology provides the opportunity for teachers to gather data in a new and different way. The digital formative assessment platform (STEP) used by researchers helped teachers in evaluating and analysing a large number of students' responses and providing feedback much more quickly than the paper-based methods.

The introduction of Interactive White Boards (IWB) provides opportunities to integrate and display an 'infinitely wide range of ready-to-use' DCR when connected with the Internet (Hennessy, 2011, p. 476 cited in Saville, Beswick, & Callingham, 2014). Teachers can use, draw, manipulate, create, display, and disseminate content through IWB, and can conduct assessments (Saville et al., 2014). Therefore, rather than focusing on traditional ways, teachers could be provided opportunities to play and explore in a small micro-domain and taking risks using alternate instructional (digital) resources. Institutions need to support and nurture these small micro-domains or creative classrooms instead of adding more courses related to technology in professional development programmes for teachers (Mishra, 2104). In this way, new knowledge and skills may be developed by teachers that may enable them to create, select and use technology-mediated new ways of representing the subject matter content knowledge, observe and evaluate student work and learning progression using technology (Tabach & Trgalová, 2019).

The above section discussed how different teachers integrated DCR for teaching mathematics. Several other studies - not discussed here - (see for example; Gueudet, Pepin, Sabra, & Trouche, 2016; Pepin, Gueudet, et al., 2017; Pepin, Choppin, et al., 2017; Trouche, Gueudet, & Pepin, 2018) have also shown the potential uses of other digital content resources such as e-textbook and the computer algebra system for mathematics education. Despite the available opportunities (as discussed above) and the potential of using DCR for mathematics education, educators and educational researchers have not been able to streamline their use in developing countries (Kalolo, 2019). In developing countries, teachers seem to be using digital technologies under institutional pressure and without the knowledge and skills that could help identify the role of a specific digital resource in the transformation of teaching and learning processes (Kalolo, 2019). Also, many teachers in developing countries (like Pakistan) have not been introduced and trained with teaching resources (such as e-textbook or GeoGebra). Most likely, when they were learners (a decade or two ago), they may not have encountered digital resources so that they may or may not realise the potential use, complexity and challenges involved in using. Therefore, the next section will discuss different types of challenges and complexities of using DCR in mathematics education.

2.4 Complexities/Challenges in using digital content resources

Studies (Akpinar & Simsek, 2007; Choppin et al., 2014; Gaffney, 2010; Ruthven, 2014) have shown that there exist technological, pedagogical and logistical challenges in the integration of DCR. According to Choppin et al. (2014), a major logistical challenge in adopting full digital curriculum materials is that many digital resources cannot be accessed without the Internet. Digital divide research shows a gap between high and low SES (social-economic status) populations in the most highly developed countries, particularly in terms of broadband access. These challenges are more complex in developing countries like Pakistan where ICT (information communication technology) penetration remains limited, i.e. only 19% of households have computers, only 24% of which have internet access (ITU, 2017). These numbers show Pakistan has still got insufficient access to the broadband services and related support to make a fair transition for the use of DCR.

One important consideration or rather a challenge for teachers is to distinguish between Open educational resources (OER) and 'restricted resources'(Trouche et al., 2018). Most of the commercial resources available over the Internet have an 'exhibit' version and not a full version. For example; most of the video content and quizzes available at the MOOC (Massive open online courses) can be accessed freely, "but require registration, and sometimes fees, for validating one's participation" (Trouche et al., 2018, p. 5). Obviously, without getting access to complete digital content for teaching, it is highly likely that teachers will find difficulties to integrate DCR into teaching and learning.

Apart from logistical challenges, studies (see Akpinar & Simsek, 2007; Choppin et al., 2014; Intel Corporation, 2011; Pepin, Choppin, et al., 2017; Ruthven, 2014) have discussed the complexities of using DCR by teachers. Akpinar and Simsek (2007) found both pre-service and in-service teachers at K-12 level struggle to use and embed basic digital assets (digital pictures, animations, simulation, sound files, hyperlink games, and video) in the design of their curriculum content. Akpinar and Simsek observed that most of the participants used just text and very less digital assets to create their teaching content. These findings are consistent with the results of the empirical study carried out in Pakistan. Nisar, Munir and Shad (2011) have found text and images presented through PowerPoint as the most common type of DCR used in classrooms. Such uses of DCR perhaps less likely to navigate the learning experiences of students in a digital environment.

Moreover, when considering the delivery of DCR, one should not underestimate the importance of having accessible, useful and reliable technological tools (Gaffney, 2010). Whatever technological tools and applications are used (for example IWB or dynamic software), they must support the desired curriculum and educational culture in schools, and teachers must have the technological skills, technical assistance and other resources to make efficient and effective use of them. (Mumtaz 2000 cited in Gaffney 2010). Therefore the following section moves on to discuss teachers' capabilities and how literature framed the aspects of developing technological capabilities of teachers.

2.5 Technological capability

Over the past decades, due to the continuously changing technological environment, different concepts have been introduced to define the term technological capabilities (Doyle, Seery, Canty, & Buckley, 2019). The Irish "National Council for Curriculum and Assessment" NCCA (2004) defined the term technological capability using a framework which state capability as the application of basic knowledge and skills through creative and sensitive thoughts and actions. NCCA (2004) also included problem-solving skills, communication skills, design and realisation skills in the framework. To develop capabilities, NCCA (2004) considered abovementioned skills vital along with the ability to think critically when evaluating artefacts, systems, and technological activities (Doyle et al., 2019). From the teachers perspective, Gaffney (2010) describes the term 'capabilities' as "teachers' potential and facility in using digital technologies" (p. 8). Teachers are considered technologically capable when they know how to transfer knowledge, skills and values using technology. They know how to develop, analyse, and change a particular technological resource, the right purpose and use of the technological resources, and how it can be integrated into teaching and learning.

In the above context, however, Tabach and Trgalová (2020) have defined the digital technological capabilities of mathematics teachers by introducing the Digital Competencies concept. They compared different international standards for integration of digital technologies in the professional development of teachers and concluded that; positive opinion about technology, personal orientation, digital skills, and knowledge of digital content, tools, curriculum, and students constitute digital competencies for teachers. Tabach and Trgalová (2020) consider mathematics teachers digitally competent when they are capable of using digital media, resources and digital tools confidently, effectively, and responsibly in teaching. In the context of this discussion where the aim is to find ways to investigate mathematics teachers' knowledge and skills used to integrate digital content resources, the term digital competencies seem more appropriate than the term capabilities. For digital competencies, Tabach and Trgalova (2020) suggest it is important for a teacher to develop digital judgement by learning skills, best strategies for the use of the Internet to design, amend, search, and disseminate digital content. In light of this discussion, it is deemed important to discuss in detail how teacher technological capabilities and competencies are developed and evaluated in the literature.

2.6 TPACK (Technological Pedagogical Content Knowledge)

To develop and evaluate teachers' technological capabilities Mishra and Koehler (2006) proposed a model. They introduce a third knowledge area (technological knowledge – TK) into Shulman (1986)'s conceptual framework of "pedagogical content knowledge" (PCK). Mishra and Koehler explained that there are three overlapping circles representing content knowledge (CK), pedagogical knowledge (PK) and technological knowledge (TK) as shown in Figure 1. The intersection of CK, PK, and TK is the representation of a special kind of teacher knowledge called *"Technological Pedagogical Content Knowledge"* (TPACK or TPCK).

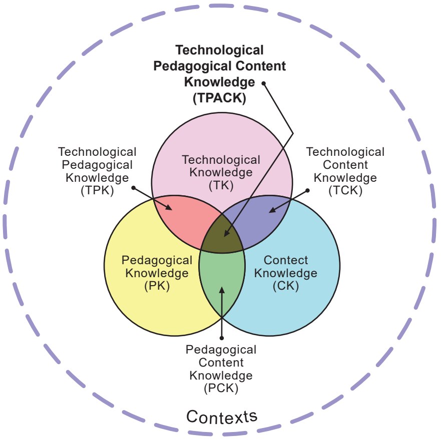

Figure 1: TPACK model as shown at http://tpack.org/

The use of TPACK for empirical studies in the literature of mathematics education has been questioned. For example, Graham (2011) criticise the TPACK framework because it relied on broad and undefined constructs. Graham argues "TPACK framework is built on PCK that lacks theoretical clarity" (p. 1955). TPACK required a more in-depth description to clarify the balance between parsimony and complexity of its constructs. Another important criticism TPACK has received is regarding its generalisability; the framework is not mathematics-specific (Koehler et al., 2007; Tabach & Trgalová, 2019). Mathematics involves a different set of complexities and content knowledge as exemplified by Ball, Thames, and Phelps (2008) in the framework called "mathematical knowledge for teaching" (MKT). Thus we require frameworks that are developed with mathematics education in mind. Studies have pointed out that to make proper links between mathematical content, pedagogy and technology, teachers are required to have strong pedagogical and content knowledge in mathematics (Clark-Wilson et al., 2014; Crompton, 2015; Niess, 2015). Teachers' mathematical knowledge in finding, linking and recognising resources to teach mathematics are deemed critical (Ball et al., 2008). Such weakness may constraint the ability of teachers to identify "solid mathematical and didactical knowledge" presented in a DCR (Aldon & Trgalova, 2019; Tabach & Trgalová, 2019). Therefore, it is critical to understand how the teacher's knowledge of teaching mathematics is defined in the literature and how this knowledge can facilitate the selection of DCR for teaching and learning of mathematics.

2.7 Mathematical Knowledge for Teaching (MKT)

Ball, Thames, and Phelps (2008) identified that teachers need *"Mathematical Knowledge for Teaching" (MKT)* that is defined as *"the mathematical knowledge needed to carry out the work of teaching mathematics"* (p. 395). MKT definition starts with teaching, not teachers which Ball et al. explained as "everything that teachers must do to support the learning of their students" (p. 395). Ball et al. qualitatively analysed the recurring tasks and different problems that are associated with the teaching and learning of mathematics. They analysed what teachers do when they teach mathematics, and what knowledge, skills, and sensibilities are required to manage such tasks. In their analysis of teacher' knowledge, they used the concept of Pedagogical Content Knowledge (PCK) presented by Shulman (1986) to introduce MKT. They explained, MKT is comprised of *"four domains of mathematical knowledge as common content knowledge (CCK), specialised content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT)"* (p. 396). Ball et al (2008) further put these domains of MKT broadly into two categories; *'subject matter knowledge'* and *'pedagogical content knowledge'* as shown in Figure 2.

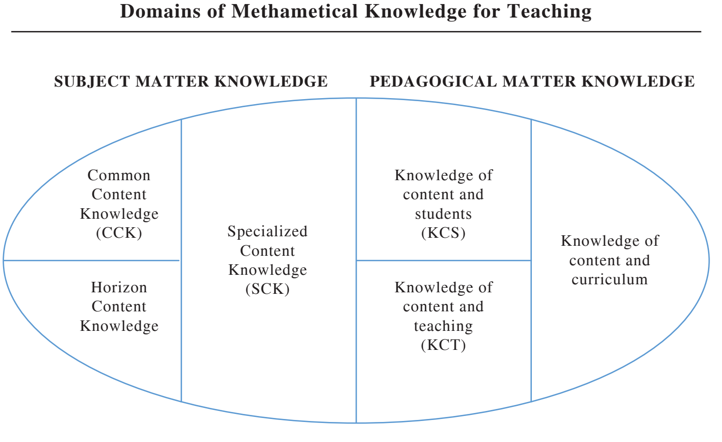

Figure 2: Mathematical Knowledge for Teaching (MKT)'s domains

Adopted from Ball et al. (2008)

Ball et al. (2008) established that mathematical tasks of teaching mainly constitute and revolves around teachers' knowledge and skills in finding, presenting, representing, linking and selecting mathematical ideas for teaching. They emphasised that teachers' MKT is culturally specific or in other words, dependent on teaching styles. However, explanation of mathematical ideas that provide sense to students is central regardless of any style of teaching.

Schoenfeld (2011) criticised MKT as it fails to consider the importance of teachers' beliefs. The importance of including beliefs in studies of teachers' knowledge has been emphasised, and some even argue for the equivalence of beliefs and knowledge. Beliefs are part of teacher orientation and goals, they are part of affective aspects and informs about "how and why teachers make the choices they make, as they teach" (Schoenfeld, 2011, p. 458). Schoenfeld (2011) emphasised on including teachers' beliefs to increase the validity of studies on teachers' knowledge of teaching mathematics. Therefore, in the context of this discussion, frameworks rooted within mathematics education which takes into account MKT along with teachers' beliefs such as PTK (Pedagogical Technology Knowledge) are required (Tabach & Trgalová, 2019). PTK take into account not only MKT but also factors such as personal and professional knowledge, personal orientation (beliefs, preferences, and values as defined by Schoenfeld (2011)) for the use of digital technology by mathematics teachers. These frameworks will be discussed in the next section because they provide conceptual guidance for understanding knowledge and skills of teachers for the use of digital technologies.

2.8 Pedagogical Technology Knowledge (PTK)

The introduction of any new technology demands that teachers also must develop the mindset that can facilitate broader perspectives about the utility of digital technology in mathematics education (Thomas & Hong, 2005). Only the knowledge of technology for the successful mathematics outcome is not enough. Teachers need to develop pedagogical technology knowledge (PTK), i.e. *"knowing how to teach mathematics with technology"* (Thomas & Palmer, 2014, p. 71). PTK develops when teachers advance through the phases of instrumentation and instrumentalisation of resources and gain a personal understanding of the role of resources in teaching and learning of mathematics. According to Trouche, Gueudet, and Pepin (2018), the instrumentation is a process that focuses on the impact of resources on the work of a teacher, while the instrumentalisation focuses on the teacher's impact on the resources he/she works on/with. The instrumentation and instrumentalisation are the two components of the concept called instrumental genesis. The theoretical basis for Instrumental genesis was originally developed by Verillon and Rabardel (1995) in the fields of cognitive ergonomics and educational psychology. Verillon and Rabardel emphasise that there is a difference between an artefact (a physical material object) and an instrument (a psychological construct). They further explained, *"an instrument does not exist in itself, it becomes an instrument when the subject has been able to adapt it to him/herself and has integrated it into his/her activity"*. The change of an artefact or tool into an instrument is called *"instrumental genesis"*, a complex process linked to the artefact's characteristics (its potentialities and limitations) and the subject's activity, knowledge and previous habits of working (Guin & Trouche, 2002).

Thomas and Hong (2005) explain PTK as a construct that includes instrumental genesis, mathematical content knowledge (MCK), MKT and personal orientation. Figure 3 shows how these three teacher-related factors are combined to produce PTK. For personal orientation, PTK uses Schoenfeld (2011) definition that emphasises on teacher's beliefs and goals about the importance of technology, the essence of learning mathematical knowledge, affordances and constraints involved, and affective aspect, i.e. how confident teacher is in the use of technology (Tabach & Trgalová, 2019, p. 190; Thomas & Hong, 2013). These foundations of PTK differentiates it from TPACK that articulates the relationship between PCK, TPK, and TCK. Koehler, Mishra and Cain (2009) explain, TPACK has no predefined goals, but it explains how classroom teachings might change by using any particular technology. Whereas, PTK takes account of strong mathematical content knowledge, teacher's personal orientation towards technology with specific predefined goals and the level of teachers' confidence in using technology (Thomas & Palmer, 2014). Therefore, a teacher with strong PTK will be more capable of demonstrating and developing true and justified knowledge of mathematical (digital) objects under study and will be able to strongly embed mathematical conceptions and understanding at the centre of classroom activity rather than technology.

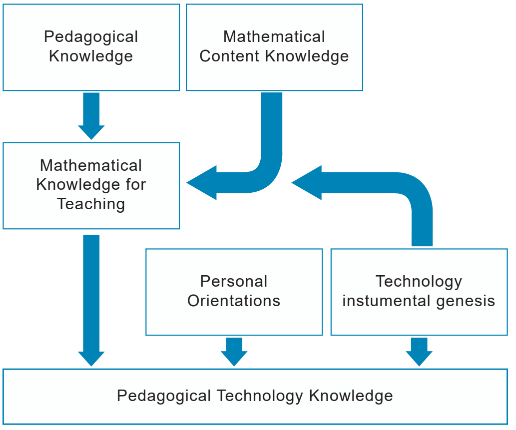

Figure 3: PTK model/framework by Thomas and Palmer (2014)

These arguments provide a critical investigative lens for this proposed research to understand the use of DCR by mathematics teachers. However, recent literature (Pepin, Gueudet, et al., 2017; Pepin, Choppin, et al., 2017; Ruthven, 2014; Tabach & Trgalová, 2019, 2020; Trouche et al., 2018; van den Bogaart et al., 2019) have more specifically addressed the use of digital content resources. Tabach and Trgalová (2019) and Pepin, Choppin, et al. (2017) have discussed standards in the use of digital content resources for mathematics education whereas van den Bogaart et al. (2019) have discussed the issues of co-design and development of digital content for mathematics. Trouche et al. (2018) proposed a theoretical frame they called *"the documentational approach to didactics"* (DAD) that has been used by researchers (see Rocha, 2018), as a tool for analysing the impact of digital content resource (e-textbook) on the design of mathematics teaching. However, Tabach and Trgalová (2020) modifications of MKT (Ball et al., 2008) and PTK (Thomas & Hong, 2005) frameworks to propose a new framework called Digital competencies for teaching mathematics with technology provide more relevant constructs such as double instrumental genesis and mathematical digital knowledge of teaching to investigate knowledge and skills of mathematics teachers using DCR. The next sections, therefore, discuss the modification brought by Tabach and Trgalová in MKT and PTK framework and how those modifications inform our discussion.

2.9 Digital competencies for teaching mathematics with technology

Tabach and Trgalová (2019) extended the previous work on PTK framework (cf. Thomas et al., 2008; Thomas & Palmer, 2014) to understand better the desired knowledge and skills of mathematics teachers for the use digital resources in classrooms. They examined different international standards describing mathematics teachers' digital technology-related knowledge by using PTK framework. They consider the use of PTK framework appropriate as the framework is specifically developed for mathematics teachers in mind. However, Tabach and Trgalová proposed two changes for the pedagogical technological knowledge (PTK) framework. These changes laid the foundation for a new framework which was initially called Mathematical knowledge for teaching with technology (MKTT) (Tabach & Trgalová, 2019).

First, instead of *"technology instrumental genesis"* Tabach and Trgalová used *"double instrumental genesis"* approach (Haspekian, 2011). Double instrumental genesis is a framework proposed by Haspekian (2011) which incorporates two instrumental geneses (personal and professional) of teachers. The framework has its theoretical foundation from Rabardel (2002) concept of instrumental genesis. According to it, "teachers must first acquire basic skills to master the specific technology they intend to use and develop utilisation schemes related to this technology (personal instrumental genesis). They must also develop their understanding of how to support students' mathematics learning in a digital environment (professional instrumental genesis)" (Tabach & Trgalová, 2019, p. 188). Haspekian (2011) regarded personal genesis is common to any teacher (although it is tool-specific) while the professional genesis is specific to teachers of mathematics. Pepin, Choppin, et al. (2017) and Ruthven (2017) proposed that most studies on digital content resources consider digital content as a 'digital mathematical tool' and can be examined under the lens of Rabardel (2002)'s instrumental approach.

Second, Tabach and Trgalová (2019) modified the original MKT framework by Ball et al. (2008) by further adding the dimension of digital technology in the four out of six domains of MKT. Tabach and Trgalová refer to it as Mathematical Digital Knowledge for Teaching (MDKT) and defined each component of MDKT, as shown in Figure 4.

Specialized Digital Content knowledge (SDCK)	Teachers' specialized digital content knowledge (SDCK) with respect to the mathematics to be taught
Knowledge of Digital content and students (KDCS)	Knowledge og content and students, which in a technological environment includes additional aspects that may be formulated as knowledge of digital content and students
Knowledge of Digital content and teaching (KDCT)	Knowledge og content and teaching, which in a technological environment may be interpreted as knowledge of digital content and teaching
Knowledge of Digital content and curriculum (KDCC)	Knowledge of content and curriculum in a digital environment, e.g., knowledge of prescribed use of ICT

Figure 4: Definition of each component of MDKT by (Tabach & Trgalová, 2020)

The above modification in the PTK framework, i.e. introducing double instrumental genesis and adding a dimension of digital technology in MKT (Ball et al. 2008) leads to a new framework called "digital competencies for teaching mathematics with technology" as shown in Figure 5.

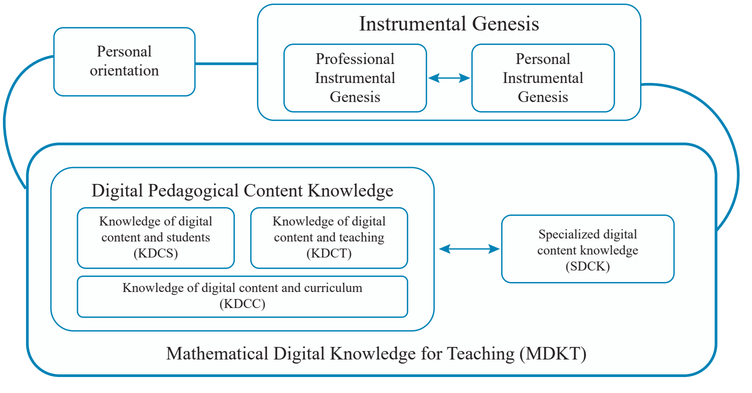

Figure 5: *"Digital competencies for teaching mathematics with technology"* framework (Tabach & Trgalová, 2020)

Thus the above framework (Figure 5) comprises three domains: teachers' orientations (belief, attitude, and preferences), teachers' (digital) knowledge, and teachers' double instrumental genesis related to digital technology (Tabach & Trgalová, 2019, 2020). Schoenfeld (2011) consider teacher personal orientations and goals are very important when using any specific technology or content for teaching. These orientations and goals are "related to the affective domain and teachers' beliefs regarding mathematics, teaching mathematics and digital technology". Other studies (cf. Ruthven, 2014; Tabach & Trgalová, 2019; Thomas & Hong, 2013; Thomas & Palmer, 2014) have also mentioned the impact of teachers' orientations and goals on the confidence of using any digital resources for teaching. Schoenfeld (2011) considered teachers' positive attitude and specific goals towards technology are essential when integrating digital technologies.

Tabach and Trgalová (2020) framework defining digital competencies of mathematics teachers can be used to study the integration of DCR. The framework is built on MDKT that define four domains of teachers' digital knowledge (SDCK, KDCS, KDCT and KDCC). In these four domains, SDCK (specialised digital content knowledge) is closely linked to a teacher's personal instrumental genesis. Whereas the other three domains KDCS, KDCT, and KDCC *"are linked to the professional instrumental genesis, the student instrumental genesis, and the genesis of learning mathematics with digital technology"* (Tabach & Trgalová, 2019, p. 189). These domains are widely used and mentioned in the various international policy documents to define ICT-related digital competencies, knowledge and skills of mathematics teachers, such as the *"EU DigComp framework"* and *"Australian national framework for professional standards for teaching"* which also included skills of searching and identifying relevant digital resources in different online repositories as part of digital competencies. These documents also emphasised on teachers' ability to design and develop digital resources themselves or with the help of peers, and teachers' ability to share digital resources with other teachers and their students also part of teachers' digital competencies (Tabach & Trgalová, 2020). These essential aspects of teachers' abilities are captured by the Digital competencies framework as teacher professional instrumental genesis. Whereas, mathematics teachers' knowledge about DCR and of students in a technological environment are captured using KDCS and KDCC.

3. Discussion

The above discussion identified relevant frameworks for investigating and guiding mathematics' teaching using technology such as TPACK (Koehler et al., 2009), PTK (Thomas & Hong, 2013; Thomas & Palmer, 2014), Double Instrumental genesis (Haspekian, 2011), MKT (Ball et al., 2008), MDKT (Tabach & Trgalová, 2019) and Digital competences framework (Tabach & Trgalová, 2020). In this discussion, the important point made was about the generalizability of TPACK, as researchers in mathematics education have argued that mathematics teaching required a different set of skills gained by performing different tasks of mathematics as elaborated in MKT by Ball et al. (2008). Therefore, PTK, which includes MKT, is more relevant when investigating the teachers' use of technology in the mathematics classroom. PTK also include beliefs of teachers about the importance of technology, their personal orientation and process of mastering the technology through the process of instrumental genesis. These factors are not discussed in other frameworks, such as TPACK. The literature related to PTK argues that the development of teachers' personal instrumental genesis is essential before using it in the classroom. However, the issue of whether these two geneses can be developed simultaneously or sequentially is still argued in the literature (Sacristán, 2019). To address digital competence and skills of mathematics teachers which they employ in the identification of digital resources, the literature identified the Digital Competencies framework, which builds on the foundation of both the MKT and PTK framework. Digital Competencies framework may be used as a lens to observe knowledge and skills that are expected of mathematics teachers in Pakistan when using/selecting/up taking digital content resources for teaching and learning of mathematics.

The discussion also identified some gaps. For instance, the digital competencies framework is only being used to study policy documents (see for example; Tabach & Trgalová, 2019, 2020), and it is proposed that it be used empirically to evaluate the ICT competencies of teachers, which involves the use of digital content resources (DCR) for teaching mathematics. Also, studies in the field have presented findings from the context of developed countries such as USA, UK, and other European countries. These countries are reasonably advanced in the use of digital technologies, paying great attention to the development of technological infrastructure and the acquisition of new tools for teaching and learning. As a result, they do possess a sophisticated and well-established system for the professional development of teachers as compared to Pakistan. Therefore, the generalisability of these studies is questionable when used in the context of developing country high school mathematics classrooms.

Moreover, in Pakistan, it is only recently that the digital content resources designed and developed locally are becoming available. For example, the websites www.sabaq.pk (Sabaq Foundation), www.maktab.pk (Maktab), www.elearn.punjab.gov.pk (eLearn Punjab) and www.gooverdesk.com (DESK) have locally developed bilingual digital content resources in the shape of videos, e-textbooks and online assessments. The quality, relevance and awareness about these resources are still questionable. Though teachers can use the local DCR specific to local curricula, people who create such digital resources do not provide information about how to integrate them into mathematics education. In Pakistan, limited research has been undertaken to investigate the factors that influence teachers to integrate digital content resources at the high school level. Most studies evaluating the use of digital technologies in mathematics education are either purely qualitative or quantitative, very few have used mixed-methods but in a different context. These gaps provide an opportunity to study the integration of DCR in Pakistani high school mathematics classrooms using different methods to support learning and teaching of mathematics. Most importantly, in the current global COVID-19 crisis where education around the globe is moved online, an unprecedented urgency is required to provide high-quality digital content to help students, teachers and even parents who are currently at home locked-down to continue teaching and learning. This urgency is not only limited to the teachers of the developed world, every teacher now supposed to have knowledge and skills to critically, creatively and confidently use digital tools and resources to achieve personal and professional goals.

4. Conclusion

The integration of DCR into mathematics' teaching is a process exclusive to each teacher. Teachers with strong/weak technology or content knowledge can take a different path when choosing DCR for teaching than those who do not. Some teachers are only users of DCR (who know how to search, select, and adapt available DCR) and some teachers are both; users and designers (who know how to (re)-design, amend, develop and share available DCR). Although studies that investigated teachers' use of digital technology in mathematics classrooms have created new knowledge, diverse models, and frameworks. However, these studies are not based on or derived from the context of developing countries. There are still knowledge gaps in teachers and students' individual use of digital content resources and required multi-dimensional, cross-disciplined, and cultural-contextual investigations. Significantly, the study of mathematics teachers' resources which include digital content resources is a newly emerging research field in mathematics education (Fan et al., 2018). The recent interest of mathematics community in the area of teachers' resources can be witnessed by the number of related articles presented in the major four-yearly International Congresses of Mathematics Educators ICME-12 (2012) and ICME-13 (2017). The most recent ICME-13 (2017) was the first to dedicate a complete Topic Study Group to the theme of teachers' resources in which multiple studies were presented on different aspects related to DCR such as quality, relevance, the time required for integration of DCR, and knowledge and skills on part of teachers (Trouche & Fan, 2018). Therefore, it is presumed that the research using an investigative lens of frameworks such as TPACK, PTK, and Digital Competencies for teaching mathematics with technology may attract the mathematics research community in Pakistan. It may also attract educational policymakers, academia and students both at national and international levels. Such studies may also help institutions in understanding their role which they need to play designing and developing mathematics curriculum, professional development programme for teachers, acquiring technological infrastructure, resources, and employing modern digital inspection regime.

References:

Akhter, N., Akhtar, M., & Abaidullah, M. (2015). The Perceptions of High School Mathematics Problem Solving Teaching Methods in Mathematics Education. Bulletin of Education and Research, 37(1), 55–77.

Akpinar, Y., & Simsek, H. (2007). Should K-12 Teachers Develop Learning Objects ? Informing Science and IT Education Joint Conference, 6. Istanbul, Turkey: Informing Science Institute.

Aldon, G., & Trgalova, J. (2019). Technology in Mathematics Teaching (Volume 13). https://doi.org/https://doi.org/10.1007/978-3-030-19741-4 ©

Arzarello, F., & Sabena, C. (2014). Introduction to the Approach of Action, Production, and Communication (APC) BT - Networking of Theories as a Research Practice in Mathematics Education (A. Bikner-Ahsbahs & S. Prediger, Eds.). https://doi.org/10.1007/978-3-319-05389-9_3

Aslam, M., & Kingdon, G. (2011). What can teachers do to raise pupil achievement? Economics of Education Review, 30(3), 559–574. https://doi.org/10.1016/j.econedurev.2011.01.001

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching : What Makes It Special? Journal of Teacher Education, 59(5), 389–407. https://doi.org/10.1177/0022487108324554

Choppin, J., Carsons, C., Bory, Z., & Cerosaletti, C. (2014). A Typology for Analyzing Digital Curricula in Mathematics Education. International Journal of Education in Mathematics, Science and Technology, 2(1). https://doi.org/10.18404/ijemst.95334

Clark-Wilson, A., Aldon, G., Cusi, A., Goos, M., Haspekian, M., Robutti, O., … Thomas, M. (2014). The challenges of teaching mathematics with digital technologies. Proceedings of the Joint Meeting of PME 38 and PME-NA 36, 1, 87–116.

Crompton, H. (2015). Pre-service Teachers' Developing Technological Pedagogical Content Knowledge (TPACK) and Beliefs on the Use of Technology in the K-12 Mathematics Classroom: A Review of the Literature BT - Technological Pedagogical Content Knowledge: Exploring, Developing. In C. Angeli & N. Valanides (Eds.), Technological Pedagogical Content Knowledge Exploring, Developing, and Assessing TPCK (pp. 239–250). https://doi.org/10.1007/978-1-4899-8080-9_12

de Araujo, Z., Otten, S., & Birisci, S. (2017). Teacher-created videos in a flipped mathematics class: digital curriculum materials or lesson enactments? ZDM - Mathematics Education, 49(5), 687–699. https://doi.org/10.1007/s11858-017-0872-6

Doyle, A., Seery, N., Canty, D., & Buckley, J. (2019). Agendas, influences, and capability: Perspectives on practice in design and technology education. International Journal of Technology and Design Education, 29(1), 143–159. https://doi.org/10.1007/s10798-017-9433-0

Esguerra-Prieto, B., González-Garzón, N., & Acosta-López, A. (2018). Herramientas de software matemático para la enseñanza de números complejos. Revista Facultad de Ingeniería, 27(48), 79–90. https://doi.org/10.19053/01211129.v27.n48.2018.8403

Fan, L., Trouche, L., Qi, C., Rezat, S., & Visnovska, J. E. (2018). Research on Mathematics Textbooks and Teachers' Resources (ICME 13). https://doi.org/10.1007/978-3-319-73253-4

Gaffney, M. (2010). Enhancing Teachers' Take-up of Digital Content: Factors and Design Principles in Technology Adoption (p. 28). p. 28. Ministerial Council for Education Australia.

Graham, C. R. (2011). Theoretical considerations for understanding technological pedagogical content knowledge (TPACK). Computers and Education, 57(3), 1953–1960. https://doi.org/10.1016/j.compedu.2011.04.010

Gueudet, G. (2015). Resources at the Core of Mathematics Teachers' Work BT -. In S. J. Cho (Ed.), Selected Regular Lectures from the 12th International Congress on Mathematical Education (pp. 235–251). https://doi.org/10.1007/978-3-319-17187-6_14

Gueudet, G., Pepin, B., Sabra, H., & Trouche, L. (2016). Collective design of an e-textbook: teachers' collective documentation. Journal of Mathematics Teacher Education, 19(2–3), 187–203. https://doi.org/10.1007/s10857-015-9331-x

Guin, D., & Trouche, L. (2002). Umgang der LehrerInnen mit der instrumentalen Genese in CAS-Umgebungen: Notwendigkeit von instrumentalen Orchestrierungen. ZDM - International Journal on Mathematics Education, 34(5), 204–211. https://doi.org/10.1007/BF02655823

Haspekian, M. (2011). the Co-Construction of a Mathematical and a Didactical Instrument. Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education (Cerme 7), (February), 2298–2307.

Herrington, J., Herrington, A., Mantei, J., Olney, I. W., & Ferry, B. (2009). New technologies, new pedagogies: Mobile learning in higher education. Retrieved from http://ro.uow.edu.au/

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. Cognition and Instruction, 26(4), 430–511. https://doi.org/10.1080/07370000802177235

Intel Corporation. (2011). Digital Content in the K-12 Classroom. Educause Review, 20036(202). Retrieved from http://www.intel.com/Assets/PDF/whitepaper/Digital_Content_in_the_K-12_Classroom.pdf

ITU. (2017). Measuring the Information Society Report 2017 (Vol. 1). Retrieved from https://www.itu.int/en/ITU-D/Statistics/Documents/publications/misr2017/MISR2017_Volume1.pdf

Kalolo, J. F. (2019). Digital revolution and its impact on education systems in developing countries. Education and Information Technologies, 24(1), 345–358. https://doi.org/10.1007/s10639-018-9778-3

Kanwal, F., Rehman, M., Bashir, K., & Qureshi, U. (2017). Critical Factors of E-Learning Adoption and Acceptance in Pakistan : A Literature Review. Engineering, Technology & Applied Science Research, 7(4), 1888–1893.

Keller, B. A., Hart, E. W., & Martin, W. G. (2001). Illuminating NCTM's Principles and Standards for School Mathematics. School Science and Mathematics, 101(6), 292–304. https://doi.org/10.1111/j.1949-8594.2001.tb17960.x

Khalil, M. (2016). Effect of GeoGebra (DGS) aided instructions on grade-12 students' mathematical thinking and mathematical achievement in analytical geometry (Northern University, Nowshera). Retrieved from http://prr.hec.gov.pk/jspui/handle/123456789//7260

Koehler, M. J., Mishra, P., & Cain, W. (2009). What is Technological Pedagogical Content Knowledge (TPACK)? Contemporary Issues in Technology and Teacher Education, 9(1), 60–70. https://doi.org/10.1177/002205741319300303

Lee, H., & Hollebrands, K. (2008). Preparing to teach mathematics with technology: An integrated approach to developing technological pedagogical content knowledge. Contemporary Issues in Technology and Teacher Education, 8(4), 326–341. Retrieved from http://www.editlib.org/p/28191

Maher, C., Palius, M., Maher, J., Hmelo-Silver, C., & Sigley, R. (2014). Teachers Can Learn to Attend to Students' Reasoning Using Videos as a Tool. Issues in Teacher Education, 23(1), 31–47.

Maria, A., Manuel, J., Santos, D., & Santos, D. (2015). Using GeoGebra To Study Complex Functions. 12th International Conference on Technology in Mathematics Teaching ICTMT 12, 2013–2015.

Mishra, P., & Koehler, M. J. (2006). Technological Pedagogical Content Knowledge: A Framework for Teacher Knowledge. Teachers College Record, Vol. 108, pp. 1017–1054. https://doi.org/10.1111/j.1467-9620.2006.00684.x

Mohyuddin, R. G. (2012). Misconceptions of students in learning mathematics at primary level (University of Management and Technology Lahore). Retrieved from http://prr.hec.gov.pk/jspui/handle/123456789/11183

Mullan, E. (2011). What is Digital Content? - EContent Magazine. Retrieved July 25, 2017, from http://www.econtentmag.com/Articles/Resources/Defining-EContent/What-is-Digital-Content-79501.html

Nicholas, A. J., & Lewis, J. K. (2011). The Net Generation and E-Textbooks. International Journal of Cyber Ethics in Education, 1(3), 1–10. https://doi.org/10.4018/ijcee.2011070107

Niess, M. L. (2015). Transforming Teachers' Knowledge: Learning Trajectories for Advancing Teacher Education for Teaching with Technology. In C. Angeli & N. Valanides (Eds.), Technological Pedagogical Content Knowledge: Exploring, Developing, and Assessing TPCK (pp. 19–37). https://doi.org/10.1007/978-1-4899-8080-9

Nordin, N. M., Zaman, H. B., & Din, R. (2005). Integrating Pedagogy and Instructional Design in the e-Learning Approach for the Teaching of Mathematics. International Journal of the Computer, the Internet and Management, 4(7), 9.1-9.7.

Pepin, B., Choppin, J., Ruthven, K., & Sinclair, N. (2017). Digital curriculum resources in mathematics education: foundations for change. ZDM - Mathematics Education, 49(5), 645–661. https://doi.org/10.1007/s11858-017-0879-z

Pepin, B., Gueudet, G., & Trouche, L. (2017). Refining teacher design capacity: Mathematics teachers' interactions with digital curriculum resources. ZDM - Mathematics Education, 49(5), 799–812. https://doi.org/10.1007/s11858-017-0870-8

Rabardel, P. (2002). People and technology: a cognitive approach to contemporary instruments. (Heidi Wood). université Paris.

Raza, M. (2016). The Effect of Interactive Sessions on the Academic Performance of Students in E-Learning Institutions of Pakistan. Pakistan Journal of Distance & Online Learning, II(I), 47–56. Retrieved from https://www.researchgate.net/publication/309409316

Rocha, K. de M. (2018). Uses of Online Resources and Documentational Trajectories: The Case of Sésamath. In L. Fan, L. Trouche, C. Qi, S. Rezat, & J. Visnovska (Eds.), Research on Mathematics Textbooks and Teachers' Resources: Advances and Issues (pp. 235–258). https://doi.org/10.1007/978-3-319-73253-4_11

Ruthven, K. (2014). Frameworks for Analysing the Expertise That Underpins Successful Integration of Digital Technologies into Everyday Teaching Practice. The Mathematics Teachers in the Digital Era, (April), 373–393. https://doi.org/10.1007/978-94-007-4638-1_16

Sacristán, A. I. (2019). Mathematics Teachers' Education for Technological Integration: Necessary Knowledge and Possible Online Means for its Development. Introduction to the Section. Technology in Mathematics Teaching: Selected Papers of the 13th ICTMT Conference (Vol. 13, p. 173), 173. Springer.

Saville, M., Beswick, K., & Callingham, R. (2014). The Use of Interactive Whiteboards in Education. In N. Fitzallen, R. Reaburn, & S. Fan (Eds.), The Future of Educational Research: Perspectives from Beginning Researchers (pp. 203–216). https://doi.org/10.1007/978-94-6209-512-0_17

Schoenfeld, A. H. (2011). Toward professional development for teachers grounded in a theory of decision making. ZDM - International Journal on Mathematics Education, 43(4), 457–469. https://doi.org/10.1007/s11858-011-0307-8

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. Educational Researcher, 15(2), 4–14. Retrieved from https://journals-sagepub-com.ezproxy.waikato.ac.nz/doi/pdf/10.3102/0013189X015002004

Tabach, M., & Trgalová, J. (2019). The Knowledge and Skills that Mathematics Teachers Need for ICT Integration: The Issue of Standards BT - Technology in Mathematics Teaching: Selected Papers of the 13th ICTMT Conference. In G. Aldon & J. Trgalová (Eds.), Technology in Mathematics Education (pp. 183–203). https://doi.org/10.1007/978-3-030-19741-4_8

Tabach, M., & Trgalová, J. (2020). Teaching Mathematics in the Digital Era: Standards and Beyond. In Y. Ben-David Kolikant, D. Martinovic, & M. Milner-Bolotin (Eds.), STEM Teachers and Teaching in the Digital Era: Professional Expectations and Advancement in the 21st Century Schools (pp. 221–242). https://doi.org/10.1007/978-3-030-29396-3_12

Thomas, M. O. J., & Hong, Y. Y. (2005). Learning Mathematics with CAS Calculators : Integration and Partnership Issues. The Journal of Educational Research in Mathematics, 15(2), 215–232.

Thomas, M. O. J., & Hong, Y. Y. (2013). Teacher integration of technology into mathematics learning. International Journal for Technology in Mathematics Education, 20(2), 69–84.

Thomas, M. O. J., & Palmer, J. M. (2014). Teaching with Digital Technology: Obstacles and Opportunities BT - The Mathematics Teacher in the Digital Era: An International Perspective on Technology Focused Professional Development. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), The Mathematics Teacher in the Digital Era (pp. 71–89). https://doi.org/10.1007/978-94-007-4638-1_4

Trouche, L., & Fan, L. (2018). Mathematics textbooks and teachers' resources: a broad area of research in mathematics education to be developed. In Research on Mathematics Textbooks and Teachers' Resources (ICME 13, p. 385). https://doi.org/10.1007/978-3-319-73253-4

Trouche, L., Gueudet, G., & Pepin, B. (2018). Open Educational Resources: A Chance for Opening Mathematics Teachers' Resource Systems? In L. Fan, L. Trouche, C. Qi, S. Rezat, & J. Visnovska (Eds.), Research on Mathematics Textbooks and Teachers' Resources: Advances and Issues (pp. 3–27). https://doi.org/10.1007/978-3-319-73253-4_1

van den Bogaart, T., Drijvers, P., & Tolboom, J. (2019). Co-Design and Use of Open Online Materials for Mathematics and Science Didactics Courses in Teacher Education: Product and Process BT - Technology in Mathematics Teaching: Selected Papers of the 13th ICTMT Conference (G. Aldon & J. Trgalová, Eds.). https://doi.org/10.1007/978-3-030-19741-4_9

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of though in relation to instrumented activity. European Journal of Psychology of Education, 10(1), 77. https://doi.org/10.1007/BF03172796